Appendix A

The JC 2 algorithm computes $C_{[j],k}$, $S_{[j+1],k}$ and $ts\_start([j+1],k)$ values for $2 \leq j+1 < len(k)$ and $1 \leq k \leq m$ for the jobs [j] and [j+1] that are processed on machine k if $DR_{[j],[j+1]} \geq 0$.

JC 2 Algorithm:

Input: λ, μ, $n_{[j]}$, $n_{[j+1]}$, $S_{[j],k}$, $ts\_start([j],k)$, $ts\_steady([j],k)$, $p_i^{[j]}$, $R_i^{[j]}$ with $i = 1,\ldots, n_{[j]}$, and $p_h^{[j+1]}$ with $h = 1,\ldots, n_{[j+1]}$

1. During cycle $\upsilon_{1,[j]}$:

1.1 $Z_0 := S_{[j],k} + ts\_start([j],k) + ts\_steady([j],k)$

1.2 $U_{1,n_{[j]}}^{[j]} \coloneqq Z_0 + \max\left\{\mu, R_{n_{[j]}}^{[j]}\right\}$, $L_{1,n_{[j]}+1}^{[j]} \coloneqq U_{1,n_{[j]}}^{[j]} + Q$

1.3 For $i = n_{[j]} - 1, \ldots, 1$:

1.3.1 $U_{1,i}^{[j]} := \max\{L_{1,i+2}^{[j]} + \mu, Z_0 + R_i^{[j]}\}$

1.3.2 $L_{1,i+1}^{[j]} \coloneqq U_{1,i}^{[j]} + Q$

2. During cycle $\upsilon_{e,[j]}$, $2 \leq e \leq DR_{[j],[j+1]} + 1$:

2.1 $U_{e,n_{[j]}}^{[j]} \coloneqq \max\left\{L_{e-1,e}^{[j]} + \mu, L_{e-1,n_{[j]}}^{[j]} + p_{n_{[j]}}^{[j]}\right\}$, $L_{e,1+n_{[j]}}^{[j]} := U_{e,n_{[j]}}^{[j]} + Q$

2.2 For $i = n_{[j]} - 1, \ldots, e$:

$U_{e,i}^{[j]} \coloneqq \left\{L_{e,i+2}^{[j]} + \mu, L_{e-1,i}^{[j]} + p_i^{[j]}\right\}$, $L_{e,i+1}^{[j]} \coloneqq U_{e,i}^{[j]} + Q$

3. During cycle $\tau_{1,[j+1]}$:

Output: $S_{[j+1],k} \coloneqq U_{1,0}^{[j+1]} = L_{DR_{[j],[j+1]}+1, DR_{[j],[j+1]}+2}^{[j]} + \mu$, $L_{1,1}^{[j+1]} \coloneqq U_{1,0}^{[j+1]} + Q$

4. If $n_{[j]} - DR_{[j],[j+1]} > n_{[j+1]}$:

4.1 During cycle $\tau_{e,[j+1]}$, $2 \leq e \leq n_{[j+1]}$:

4.1.1 $U_{e+DR_{[j],[j+1]},n_{[j]}}^{[j]} \coloneqq \max\left\{L_{e-1,1}^{[j+1]} + \mu, L_{e+DR_{[j],[j+1]}-1,n_{[j]}}^{[j]} + p_{n_{[j]}}^{[j]}\right\}$, $L_{e+DR_{[j],[j+1]},1+n_{[j]}}^{[j]} \coloneqq U_{e+DR_{[j],[j+1]},n_{[j]}}^{[j]} + Q$

4.1.2 For $i = n_{[j]} - 1,\ldots, e + DR_{[j],[j+1]}$:

$U_{e+DR_{[j],[j+1]},i}^{[j]} \coloneqq \max\left\{L_{e+DR_{[j],[j+1]},i+2}^{[j]} + \mu, L_{e+DR_{[j],[j+1]}-1,i}^{[j]} + p_i^{[j]}\right\}$, $L_{e+DR_{[j],[j+1]},i+1}^{[j]} \coloneqq U_{e+DR_{[j],[j+1]},i}^{[j]} + Q$

4.1.3 $U_{e,e-1}^{[j+1]} := \max\left\{L_{e+DR_{[j],[j+1]},e+DR_{[j],[j+1]}+1}^{[j]} + \mu, L_{e-1,e-1}^{[j+1]} + p_{e-1}^{[j+1]}\right\}$, $L_{e,e}^{[j+1]} \coloneqq U_{e,e-1}^{[j+1]} + Q$

4.1.4 For $i = e - 2,\ldots, 0$:

$U_{e,i}^{[j+1]} := \max\{L_{e,i+2}^{[j+1]} + \mu, L_{e-1,i}^{[j+1]} + p_i^{[j+1]}\}$ for $i > 0$, otherwise $U_{e,i}^{[j+1]} := L_{e,i+2}^{[j+1]} + \mu$, in both situations $L_{e,i+1}^{[j+1]} := U_{e,i}^{[j+1]} + Q$

4.2 During cycle $\tau_{e,[j+1]}$, $n_{[j+1]} + 1 \leq e \leq n_{[j]}$ - $DR_{[j],[j+1]}$:

4.2.1 Repeat the Steps 4.1.1-4.1.2

4.2.2 $U_{e,n_{[j+1]}}^{[j+1]} := \max\{L_{e+DR_{[j],[j+1]},e+DR_{[j],[j+1]}+1}^{[j]} + \mu, L_{e-1,n_{[j+1]}}^{[j+1]} + p_{n_{[j+1]}}^{[j+1]}\}$, $L_{e,1+n_{[j+1]}}^{[j+1]} := U_{e,n_{[j+1]}}^{[j+1]} + Q$

4.2.3 For $i = n_{[j+1]}$ - 1, …, 0:

$U_{e,i}^{[j+1]} := \max\{L_{e,i+2}^{[j+1]} + \mu; L_{e-1,i}^{[j+1]} + p_i^{[j+1]}\}$ if $i > 0$, otherwise $U_{e,i}^{[j+1]} := L_{e,i+2}^{[j+1]} + \mu$, in both situations, $L_{e,i+1}^{[j+1]} := U_{e,i}^{[j+1]} + Q$

4.3 **Output:** $C_{[j],k} := L_{n_{[j]},1+n_{[j]}}^{[j]}$, $ts\_start([j+1],k) := L_{n_{[j]}-DR_{[j],[j+1]},1}^{[j+1]}$ - $S_{[j+1],k}$

5. Otherwise if $n_{[j]}$ - $DR_{[j],[j+1]} \leq n_{[j+1]}$:

5.1 During cycle $\tau_{e,[j+1]}$, $2 \leq e \leq n_{[j]}$ - $DR_{[j],[j+1]}$, repeat the Steps 4.1.1-4.1.4

5.2 During cycle $\tau_{e,[j+1]}$, $n_{[j]} - DR_{[j],[j+1]} + 1 \leq e \leq n_{[j+1]}$:

5.2.1 $U_{e,e-1}^{[j+1]} := \max\{L_{e-1,1}^{[j+1]} + \mu, L_{e-1,e-1}^{[j+1]} + p_{e-1}^{[j+1]}\}$ if $e > n_{[j]} - DR_{[j],[j+1]} + 1$, otherwise

$U_{e,e-1}^{[j+1]} := \{L_{n_{[j]},n_{[j]}+1}^{[j]} + \mu, L_{e-1,e-1}^{[j+1]} + p_{e-1}^{[j+1]}\}$, in both situations $L_{e,e}^{[j+1]} := U_{e,e-1}^{[j+1]} + Q$

5.2.2 For $i = e$ - 2,…,0:

$U_{e,i}^{[j+1]} := \max\{L_{e,i+2}^{[j+1]} + \mu; L_{e-1,i}^{[j+1]} + p_i^{[j+1]}\}$ if $i > 0$, otherwise $U_{e,i}^{[j+1]} := L_{e,i+2}^{[j+1]} + \mu$, in both situations $L_{e,i+1}^{[j+1]} := U_{e,i}^{[j+1]} + Q$

6. **Output:** $C_{[j],k} = L_{n_{[j]},1+n_{[j]}}^{[j]}$, $ts\_start([j+1],k) = L_{n_{[j+1]},1}^{[j+1]}$ - $S_{[j+1],k}$.

The starting time of the close-down period of job [*j*] is $Z_0 := S_{[j],k} + ts\_start([j],k)$ + $ts\_steady([j],k)$. By the BESS algorithm we know that starting from Z_0, $\upsilon_{1,[j]}$ will be executed. Thus, the robot first moves to the $(n_{[j]})^{\text{th}}$ step and the value of $U_{1,n_{[j]}}^{[j]}$ depends on if the wafer in this step is finished or not. Hence, we have Statement 1.2. Then, the robot moves to step i, $i = n_{[j]}$ - 1, $n_{[j]}$ - 2, …, 1, unloads a wafer there, moves to step i + 1 and loads the wafer there. Thus, Statement 1.3 is realized in a similar way.

During cycle $\upsilon_{e,[j]}$, $2 \leq e \leq DR_{[j],[j+1]} + 1$, job [*j*] is performed in a decremental way and job [*j*+1] has not started. Therefore, Statements 2.1 and 2.2 can be obtained easily.

During cycle $\tau_{1,[j+1]}$, as we can see from Fig. 7, we only unloads the first wafer of job [*j*+1] from one load lock and loads it into the first step of $c_{[j+1]}$. Combined with Statement 2.2 we get Statement 3.1.

In the following, if $n_{[j]}$ - $DiR_{[j],[j+1]} > n_{[j+1]}$, job [*j*+1] reaches the steady state before *BESS_close*{[*j*]} ends. During cycle $\tau_{e,[j+1]}$, $2 \leq e \leq n_{[j+1]}$, as job [*j*] is performed in a decremental way, and job [*j*+1] is executed in an incremental way. Thus, similar to Statements 5.1.1-5.1.4 we can get Statements 4.1.1-4.1.4 in the JC 2 algorithm. Note that as $\upsilon_{1+DiR_{[j],[j+1]},[j]}$ has executed in Statements 2.1 and 2.2, hence, the index of *U* and *L* in Statements 4.1.1 and 4.1.2 has to start from 2 + $DR_{[j],[j+1]}$.

During cycle $\tau_{e,[j+1]}$, $n_{[j+1]} + 1 \leq e \leq n_{[j]}$ - $DR_{[j],[j+1]}$, job [*j*+1] has reached the steady state but job [*j*] is still run in a decremental way. Thus, we repeat Statements 4.1.1 and 4.1.2 for job [*j*]. Combined with Statement 4.2.1, we get Statements 4.2.2 and 4.2.3 such that $C_{[j],k}$ and *ts_start*{[*j*+1], *k*} can be determined.

If $n_{[j]}$ - $DR_{[j],[j+1]} \leq n_{[j+1]}$, *BESS_close*{[*j*]} ends before job [*j*+1] reaches the steady state, and during cycle $\tau_{e,[j+1]}$, $2 \leq e \leq n_{[j]}$ - $DR_{[j],[j+1]}$, jobs [*j*] and [*j*+1] are also performed in a decremental and incremental way. Therefore, we have to repeat Statements 4.1.1-4.1.4. However, during cycle $\tau_{e,[j+1]}$, $n_{[j]}$ - $DR_{[j],[j+1]} + 1 \leq e \leq n_{[j+1]}$, *BESS_close*{[*j*]} has completed but job [*j*+1] is still run in an incremental way. Thus, Statement 5.2 is obtained in a similar way such that $C_{[j],k}$ and *ts_start*{[*j*+1], *k*} can be determined.